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Estimating the Strategic Value of Long-Term Forward Purchase Contracts Using Auction Models

John E. Parsons

ABSTRACT

We demonstrate how an auction model can be used in a traditional capital budgeting context to assign a value to the strategic advantage of long-term forward contracts. Research in the field of industrial organization has pointed to the danger of ex-post opportunistic bargaining as a motivation for the use of forward contracts in natural resources and manufactured products, but no operational procedure exists for estimating the value secured by these contracts. Arbitrage methods for valuing forward contracts assume a competitive market in which the factors creating the bargaining problem and motivating the use of long-term contracts are not present. Use of the model is illustrated in the case of take-or-pay contracts for natural gas.

1. Introduction

Over the last decade much attention has been focused upon strategic factors and information problems that influence corporate financing decisions. The results of this research have been primarily suggestive--proposing possible explanations of phenomena, but not providing specific methods for incorporating the strategic factors into quantitative valuation techniques. Quantitative models of financial variables have been developed primarily for cases of perfect competition or similar special cases in which strategic factors are not central. In this paper we demonstrate how an important model developed in the literature on information economics can be used to estimate the value of strategic factors that must be incorporated in a traditional capital budgeting problem. Specifically, we apply a model of an auction to estimate the portion of a project's value which is secured to a firm through the use of long-term forward contracts for the product of the project's operation.

Long-term forward contracts are a typical element of financing for industrial projects in which large amounts of capital must be invested up front to develop production capacity and in which the market for the firm's output consists of a small number of buyers. Firms in such an industry often make the successful negotiation of forward purchase contracts a contingency upon which their decision to install capacity depends. By doing so they incur two advantages relative to firms which foresake the use of forward contracts and which choose instead to first install a given level of capacity and to then seek buyers for their products: i) they gain the information on demand that is revealed in a market price, and ii) they are likely to negotiate a higher sale price for their products since they can avoid the ex-post bargaining problem. Long-term forward contracts, therefore, improve the efficiency of capital investment decisions both directly through the information they yield and indirectly because the firm appropriates the

marginal value of its investment decision.

These two benefits to forward contracts have been discussed in two distinct sets of literature. The informational content of forward prices has been emphasized in the literature on rational expectations equilibria of a competitive economy, where by assumption all parties are able to utilize the information embedded in the publicly observable price.[1] The strategic importance of forward contracts has been stressed in the field of industrial organization, most notably in theoretical work of Williamson (1975) and of Klein, Crawford, and Alchian (1978), and in empirical work on the US coal industry and electrical utilities by Joskow (1985).[2]

Our contribution to these two literatures is to make operational these theoretical insights in a capital budgeting context. We show how one can use an auction model to derive numerical estimates of the strategic value of forward contracts. The model integrates the value of avoiding ex-post bargaining problems with the informational value of forward contracting. By

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1. We analyze, in contrast, an imperfectly competitive market in which information is garnered only by those agents engaging in forward contracting.

2. In both Williamson (1975) and in Klein, Crawford, and Alchian (1978) the primary comparison is between long-term contracts on the one hand, and vertical integration on the other. Long-term contracts are viewed in these papers as inherently unstable or incompletely enforceable so that they suffer from the ex-post bargaining problem. Vertical integration is the alternative which permits the firm to appropriate the "quasi-rents" generated by its capacity decisions. In this paper we have changed the labels for the relevant comparison. We represent an absence of contracting, i.e. waiting to use the forthcoming spot market, as the initial condition, and we analyze long-term contracts as well specified and enforceable alternatives which permit the firm to appropriate the "quasi-rents".

developing a device for estimating the significance of these considerations for any given project, we provide a tool for making specific recommendations as to whether or not the long-term contracts should or should not be used to help finance a particular capital project.

In the finance literature long-term forward contracts have been interpreted primarily as instruments that shift an exogenous price risk from one party to another. The problem of valuing these contracts is therefore reduced to a valuation of the price risk. The techniques for analyzing the price risk component of forward contracts has been pioneered by Black (1976) and Cox, Ingersoll and Ross (1981), and has been applied by Brennan and Schwartz (1985) to the valuation of long-term contracts for the output of a mine. In these papers a stochastic process for the spot price of the commodity is postulated and the pattern of forward prices which would be consistent with this process is derived. Prices are determined in competitive markets and the relationship between spot and forward prices is determined by arbitrage. The producer cannot influence the information it obtains from the market by its decision to negotiate a forward contract, nor does it influence the expected future spot price by its decision to install capacity. In this arbitrage framework the value to a seller of the forward contract relative to a decision to sell at the anticipated spot price is zero by construction: the equilibrium forward price is defined as that price which makes this value zero. Alternatively, for long-term contracts in which the delivery price is fixed and does not correspond to the equilibrium forward price, the value of the contract is the expected sum of the difference between this contracted price and the anticipated spot price. The arbitrage framework measures the value of the exogenous price risk that is shifted using these long-term contracts, but assumes away the factors generating the ex-post bargaining problem that motivates the use of these contracts in many cases.

The critical distinction between the model used in this paper and this arbitrage method is that the model used here involves imperfectly competitive markets. The producer's capacity decision and the timing for contract negotiations affect the price received: the spot price process cannot be taken as exogenous and the forward price is not determined by arbitrage vis-a-vis the anticipated spot price process. The value to the producer of forward contracts relative to spot transactions may therefore deviate from zero: the problem is to estimate how significant this deviation may be, i.e., to determine if these strategic factors are significant. To accomplish this we explicitly recognize the impact which each player's decision will have on the outcome of the negotiations forward and spot. Unfortunately to do this we need to abstract from the factors making a valuation of the price risk important and feasible.

In the next section we present an auction model and use it to define the strategic value of the forward contract. In section 3 we use this model to estimate the strategic value of take-or-pay contracts for natural gas fields. Section 4 concludes.

2. An Auction Model of Expected Sales Revenue: Forward vs. Spot Prices

We consider the problem of the firm analyzing a traditional capital budgeting decision. The firm needs to estimate the cash flows to be received from constructing a plant or developing a natural resource deposit. A key problem is to estimate the prices at which the output can be sold. The firm can choose to sell its product under long-term forward contracts or under short-term or spot contracts and it needs to make an estimate of the prices it can expect for both of these choices. The key difference between the forward and the spot market is that the firm negotiates the contract on the forward market prior to installing capacity while it negotiates the spot sale after it

has already installed its capacity: the two possible sequences of production and negotiation are illustrated in Figure 1.

[Insert Figure 1]

Since both markets are imperfectly competitive the installation of capacity substantially alters the strategic factors influencing the price. As the industrial organization literature has pointed out, given the same overall demand conditions, the seller should expect that if it first installs capacity and then negotiates the sales it will obtain a lower price on average than it would obtain in negotiations on forward contracts. The problem is to estimate this difference and its consequences for the ultimate value of the project. It is also important to estimate the value of the information on actual demand derived from the negotiations and used to determine if the capacity should be installed at all.

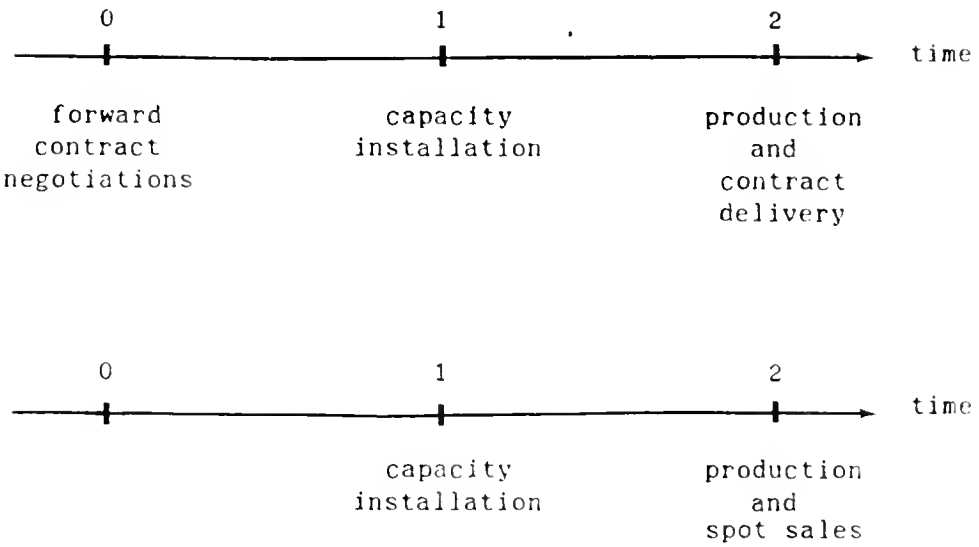
The estimate of the value associated with these two factors is derived using a negotiation game played by the producer and its potential buyers in which the payoff structure and therefore the expected outcome of the game is altered by the decision to install capacity. When this negotiation game is played prior to the installation of capacity then it yields one probability distribution of likely outcomes--prices and quantity of sales under forward contracts. When this negotiation game is played subsequent to the installation of capacity it yields another probability distribution of likely outcomes--spot prices and sales. The difference in expected profits from the two negotiation games is the estimate of the project value that is secured through the use of forward contracts as compared with spot sales.

2.1 The Auction Model

In this paper we use an auction model as an analog for the negotiation game. The auction model we use is a simple adaptation of the modified Vickrey auction analyzed in Harris and Raviv (1981). This auction is a variation on

Figure 1

Timing of Decisions under Forward Contracting
and under Production for Spot Sales.



the English or competitive/uniform-price auction. Since the price in this auction is the same for all buyers and is set close to the reservation value of the marginal buyer this auction is intuitively analogous to a competitive market. It also corresponds to our notion of a competitive market in the sense that as the number of buyers in the auction grows, the results of the auction approach the results of a perfectly competitive market.

An auction model is a well defined analog for the negotiation process. The auction model we use has certain attractive properties relative to the class of possible rules for sale: 1) it is ex-post efficient--that is the buyers who value the product most receive it: 2) it is time consistent in the sense that the seller does not close the negotiations and leave capacity unused when there exist buyers willing to purchase the product at a price greater than marginal cost: and 3) among the class of selling mechanisms which satisfy the time consistency property it is the one which maximizes the seller's expected revenue. The auction model is also advantageous for our purposes since the bidding rules and equilibrium strategies which yield the reduced form results of the auction have been derived with explicit attention to the strategic relationship among the buyers and between the buyers and the seller: this is important since the critical factor distinguishing our analysis of forward contracts from the arbitrage method is that the markets in which the commodities are to be sold are imperfectly competitive and the seller's decision to install capacity affects the future equilibrium spot price through its effect on bargaining power in the future negotiations.

To define the model we first explain the structure of the environment for which it is applicable. We assume that the Von Neumann-Morgenstern expected utility functions for the N potential buyers take the simple form in which each buyer desires up to a maximum of one unit of the commodity at any price below a given reservation price, r_i, so that $u_i(q,p) = r_i \min\{q,1\} - pq$. The

reservation price for each buyer is viewed by the seller as a random variable that may take on any of a finite set of reservation values, $\{R_1,\ldots R_\ell\}$ where $R_{j+1}=R_j+\delta$, for $j=1,\ldots\ell-1$. The probability distribution over this set of reservation values may be defined arbitrarily, but we will use for our examples the uniform distribution, $H(R_j)=1/\ell$. The producer is characterized by the scalar parameter of its constant marginal cost function, c, for a quantity of production up to a maximum of Q.

In a Vickrey auction with a capacity of Q units and a constant marginal cost of production there are two key rules determining the allocation of the output and payments that result from a given set of bids: 1) the Q buyers with the highest bids above the marginal cost of production receive the commodity--when fewer than Q bids are above the marginal cost of production, only those buyers bidding above marginal cost receive a unit and some capacity is left unused, and 2) the price paid is equal either to the highest bid among those bids not accepted or to the marginal cost of production, whichever is greater. Buyers will bid based upon their realized reservation values and their assumptions about the probable bids of other buyers. In the symmetric Nash equilibrium to the Vickrey auction each buyer's strategy is to submit a bid equal to it's valuation. Consequently the equilibrium outcome can be characterized as follows: (1) the Q buyers with the highest valuations receive the commodity--when fewer than Q buyers have reservation values above the marginal cost of production, only those buyers with reservation values above the marginal cost of production receive a unit, and (2) each buyer pays a price equal to either the highest valuation among those buyers not receiving the commodity or to the marginal cost of production, whichever is greater. Price is therefore determined by competition--when demand exceeds capacity the price clears the market, otherwise price is equal to marginal cost.

This characterization of the equilibrium outcome as a function of the

realization of the N random reservation values can be formally defined as follows. Given a specific realization of the N-buyer reservation values, $(r_1,\ldots r_N)$, define $(S_1,\ldots S_N)$ as the order statistics for the reservation values, so that $S_1 \geq S_2 \geq \ldots \geq S_N$, $S_j \in (r_1,\ldots r_N)$. Then the price at which any units of the commodity are sold is

$$(1) \qquad p = \begin{cases} S_{Q+1} & S_{Q+1} > c \\ c & S_{Q+1} \leq c \end{cases}$$

and the quantity allocated to each buyer is

$$(2) \qquad q_i = \begin{cases} 1 & r_i > \max\{S_{Q+1}, c\}; \\ \gamma & r_i = S_Q = S_{Q+1} \geq c \\ 0 & r_i < \max\{S_Q, c\} \end{cases}$$

where $\gamma = [Q - |\{S_j | S_j > S_Q\}|] \div |\{S_j | S_j = S_Q\}|$, i.e. the share obtained by dividing the remaining capacity among the buyers with reservation values equal to S_Q.

In environments similar to ours but for which the range of possible reservation values is continuous, this simple Vickrey auction is optimal for the seller--that is, among the class of feasible, incentive compatible, and time consistent mechanisms it maximizes revenue. However, for an environment such as ours with a discrete range of reservation values we must alter slightly the price rule in order to guarantee revenue maximization--the modification and the reason for it is discussed in Harris and Raviv (1981):

$$(3) \qquad p_m = \begin{cases} S_{Q+1} & S_Q = S_{Q+1} \geq R_h \\ S_{Q+1} + \delta A(Q+1) & S_Q > S_{Q+1} \geq R_h \\ R_h & R_h > S_{Q+1} \end{cases}$$

for a set of values $(A_1,\ldots A_{Q-1})$ as defined in Appendix 1, $A(j)=A_1$ when $S_j = R_i$, and where $R_h = \min\{R_j | R_j \geq c\}$. This modified price function differs from the

simple Vickrey auction price function defined in (1) in that whenever the valuations of the lowest valued buyer receiving a unit and the highest valued buyer not receiving a unit are different, $S_Q > S_{Q+1}$, the price does not drop all the way to S_{Q+1}, but is set at S_{Q+1} plus a fraction of the discrete interval between reservation values, that fraction being equal to A(Q+1).

The auction rules specified in equations (2) and (3) differ from the modified Vickrey Auction defined in Harris and Raviv (1981) only in terms of the minimum bid, R_h. This modification ensures the time consistency property and does not qualitatively change the strategic nature of the auction.[3]

In Appendix 2 we provide a display of the equilibrium outcomes of this auction model for a sample set of parameters that the reader may use to familiarize him herself with the modified Vickrey Auction.

2.2 The Comparison Between Forward and Spot Sales

The difference between the prices negotiated in forward and spot sales follows from the sunk nature of the expenditures for capacity and the weakened

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3. The optimal auction for a monopolist typically includes a minimum bid above the cost of production and this is true in the modified Vickrey Auction defined in Harris and Raviv. A minimum bid in an auction with multiple bidders is strategically analogous to a take-it-or-leave-it offer made by a seller bargaining with a single buyer. The imposition of a minimum bid implies that the seller has the power to commit itself to walk away from the auction if no potential buyer is willing to make a bid as high as this minimum; it implies that the seller is able to commit itself to refuse to sell at a lower price once it is revealed that none of the potential buyers is willing to bid the minimum. If the producer has no power to commit itself, then it is clear that a minimum bid above the cost of production cannot be a feature of the auction: a negotiation or selling strategy inclusive of a minimum bid above the marginal cost of production would not be a time consistent strategy and therefore not credible. This fact would, of course, be anticipated by the buyers and impact their bidding strategies. The maximal minimum bid which is credible is the minimal reservation value greater than the marginal cost of production, and this is the minimum bid, R_h, which we utilize in our modified Vickrey Auction as defined in (2) and (3). The logic of the proofs in Harris and Raviv can be directly applied to show that the auction defined by (2) and (3) maximizes revenue across the set of feasible and incentive compatible selling mechanisms that satisfy time consistency.

bargaining position of the seller that results. The bargaining power lost by the seller enters into the final results of our modified Vickrey auction model through the definition of the minimum bid. The minimum forward price in the modified Vickrey auction is determined as the minimum reservation value greater than the constant marginal cost inclusive of capital costs:

$$R_{hf} = \min\{R_j | R_j \geq k+v\}, \qquad (4)$$

where k is the scalar parameter of the assumed constant marginal capital cost function and v is the scalar parameter of the assumed constant operating cost function. Once capacity has been installed, however, i.e. for spot sales, the minimum price will be determined as the minimum reservation value greater than the marginal operating cost alone:

$$R_{hs} = \min\{R_j | R_j \geq v\}. \qquad (5)$$

The price function for the forward sales, p_{mf}, is then the modified Vickrey price function from equation (3) where $R_h=R_{hf}$; and the price function for the spot sales, p_{ms}, is the modified Vickrey price function from equation (3) where $R_h=R_{hs}$. The quantity allocations to the buyers are accordingly the quantity allocations from equation (2) where for forward sales c=k-v and $R_h=R_{hf}$ and for spot sales c=v and $R_h=R_{hs}$.

The profit made by the producer from a forward sale of the capacity is then a simple function of the realization of the order statistics of the buyer reservation values.

$$\pi_f(S_1,\ldots,S_N) = [p_{mf}(S_1,\ldots,S_N)-(k+v)] \sum_{i=1}^{N} q_{if}(S_1,\ldots,S_N) \qquad (6)$$

and the expected profit from the decision to use a forward sale is,

$$E(\pi_f) = \sum_{(S_1,\ldots,S_N)\in\Psi} [\pi_f(S_1,\ldots,S_N)g(S_1,\ldots,S_N)] \qquad (7)$$

where Ψ is the event space of possible combinations of order statistics $(S_1,\ldots S_N)$ and $g(S_1,\ldots S_N)$ is the probability function defined on this space from the underlying probability distribution $H(\cdot)$ over the set of reservation values $\{R_1,\ldots R_N\}$.

For spot sales the profit and expected profit functions are respectively,

$$(8) \qquad \pi_s(S_1,\ldots S_N) = ([p_{ms}(S_1,\ldots S_N) - v\,] \sum_{i=1}^{N} q_{is}(S_1,\ldots S_N)) - kQ$$

and,

$$(9) \qquad E(\pi_s) = \max(0, \sum_{(S_1,\ldots S_N)\in \Psi} \pi_s(S_1,\ldots S_N)g(S_1,\ldots S_N))$$

Note that whenever the second argument in the maximum operator is negative the producer can choose to forego any installation of capacity and thereby avoid any expectation of losses.

Our measure, Γ, of the project value secured to the producer by the use of forward contracts as compared with sales on the spot market can then be defined as,

$$(10) \qquad \Gamma = E(\pi_f) - E(\pi_s).$$

The consequences of the weakened bargaining position of a seller with installed capacity can be illustrated in the following pair of diagrams. In the first diagram the probability distributions of forward and spot prices for a sample situation are displayed: the darker distribution represents the distribution of prices which are anticipated as a consequence of forward negotiations. The lighter distribution represents the distribution of prices that are anticipated as a consequence of spot sales. In the second diagram two probability distributions over unit profits inclusive of unit capital charges are displayed. The darker distribution represents the unit profits

from forward contracting, and the lighter distribution represents the unit profits from spot contracting.

[Insert Figure 2]

The literature on opportunistic bargaining yielded the insight that the distributions for spot sales would be shifted to the left. What was missing, and what the use of an auction model promises to give us is a consistent method for assessing the magnitude of this shift for a well specified environment. The model also integrates with this measure the informational value of forward contracts. In Appendix 2 we provide a numerical example using our auction model to compare the forward and spot price distributions and to calculate Γ. a review of the example will aid in understanding the determinants of Γ.

We close this section by formalizing the argument that the ex-post bargaining problem and the informational value of forward contracting imply a higher level of profits from forward contracts than from spot sales, i.e., that $\Gamma \geq 0$.

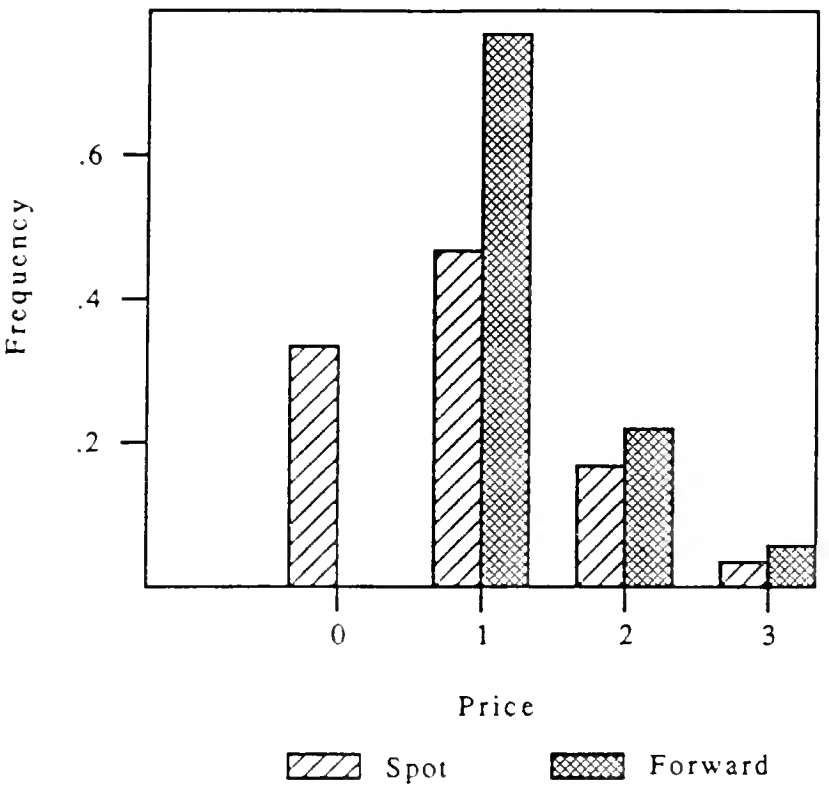
Theorem 1: $\forall \{N, Q<N, (R_1, \ldots, R_\ell), k, v, H(\cdot)\} \quad \Gamma \geq 0$, and

$\exists \{N, Q<N, (R_1, \ldots, R_\ell), k, v, H(\cdot)\}$ s.t. $\Gamma > 0$.

Proof: The first part of the proposition holds trivially when $R_{hs} = R_{hf}$ and the second holds only when $R_{hs} < R_{hf}$: we therefore assume k, v, and R such that this strict inequality holds. To establish the proposition we examine the possible cases for realizations of the two parameters S_Q and S_{Q+1} that determine the prices and quantities sold in the forward and spot auctions. Case 1: $R_{hs} < R_{hf} \leq S_{Q-1}$. Then price and quantity allocations are identical for forward and spot auctions. Case 2: $[R_{hs} \leq S_{Q-1} < R_{hf}]$ and $[S_{Q-1} < S_Q]$ and $[k+v \leq S_{Q-1}+\delta A(Q-1)]$. Then the following series of inequalities hold: $R_{hs} \leq S_{Q-1} < k+v \leq S_{Q-1}+\delta A(Q-1) = p_{ms} < S_{Q+1}-\delta = R_{hf} = p_{mf} \leq S_Q$. Since $k+v \leq p_{ms} <$

Figure 2a

Sample Probability Distribution of Forward and Spot Prices

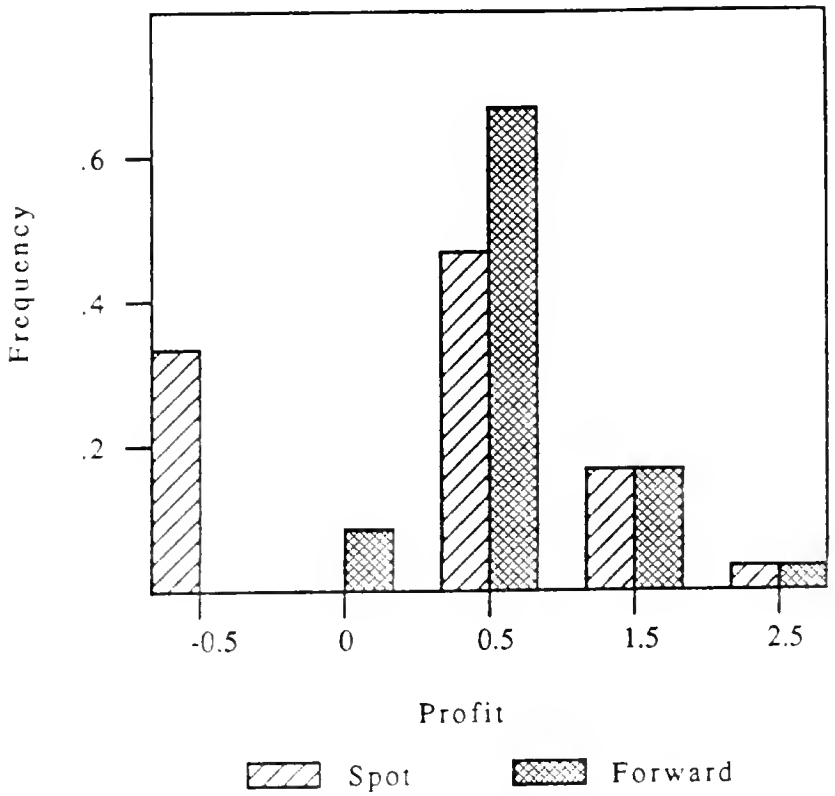


The darker distribution represents the anticipated price distribution from forward sales using the auction rules described in equations (2-4) for the parameters given in Example 2 of Appendix 2: the lighter distribution represents the anticipated price distribution for spot sales using the auction rules described in equations (2-3.5) for the same parameters. To keep the two distributions visually simple and comparable, the probability weights assigned to the modified Vickrey auction prices, $S_{Q-1}-\delta A(Q-1)$, have been reassigned to the prices S_Q and S_{Q-1} so as to maintain the same expected value.

The reasons why the distribution of spot prices is shifted leftward are as follows. In the forward contract negotiations the producer can credibly refuse a price of zero since the cost of capacity in 0.5. When conducting spot sales, and when facing two or fewer buyers with reservation values above zero, there is not adequate competition to drive the price above zero, and the producer having already installed the capacity cannot credibly refuse a price below the per unit capital charges. Hence, some sales which would occur at a price of one under forward negotiations occur at a price of zero under the spot sales.

Figure 2b

Sample Probability Distribution of Forward and Spot Unit Profits



The darker distribution represents the anticipated unit profit distribution from forward sales using the auction rules described in equation (2-4) for the parameters given in Example 2 of Appendix 2; the lighter distribution represents the anticipated unit profit distribution for spot sales using the auction rules described in equations (2-3,5) for the same parameters. Two units of capacity are installed and sometimes two units of the commodity are sold at the prices displayed in Figure 2a.

Zero profits are earned in the forward negotiations in those cases for which there are zero buyers with reservation values greater than zero and hence in which zero capacity is installed. In each of these events spot sales are made at a price of zero and profits are negative since capacity was installed ex-ante. Profits are also negative for spot sales whenever capacity was installed and, although there exist buyers with reservation values greater than zero, there does not exist competition to drive the spot price above zero.

p_{mf} and $\Sigma q_{is} = \Sigma q_{if}$ we have $0 \leq \pi_s < \pi_f$. Case 3:$[R_{hs} \leq S_{Q+1} < R_{hf}]$ and $[S_{Q+1} < S_Q]$ and $[S_{Q+1}+\delta A(Q+1) < k+v]$. Then the following series of inequalities hold: $R_{hs} \leq S_{Q+1} < S_{Q+1}+\delta A(Q+1) = p_{ms} < k+v \leq R_{hf} = p_{mf}$. Since $p_{ms} < k+v \leq p_{mf}$ we have $\pi_s < 0 \leq \pi_f$. Case 4:$[R_{hs} \leq S_{Q+1} < R_{hf}]$ and $[S_{Q+1} = S_Q]$. Then the following series of inequalities hold: $p_{ms} = S_{Q+1} = S_Q < k+v \leq R_{hf} = p_{mf}$, and therefore $\pi_s < 0 \leq \pi_f$. Case 5: $[S_{Q+1} < R_{hs} < R_{hf}]$. Then $p_{ms} = R_{hs} < k+v \leq R_{hf} = p_{ms}$ and $\pi_s < 0 \leq \pi_f$. These exhaust the possible cases. Since in every case $\pi_s \leq \pi_f$ it follows that $\Gamma \geq 0$. To establish the second part of the theorem it is sufficient to choose the parameters such that π_f is strictly greater than zero and such that one of cases 3-5 occurs with positive probability. This completes the proof.

Remarks. In case 1 where competition is great the price and quantity allocations and therefore expected profits from spot and forward sales are identical. In cases 2-5 the proof establishes that the unit operating profits are weakly greater for forward sales than they are for spot sales. This is true since the minimum forward price is greater than the minimum spot price. Of course there will be some events in which the producer would sell a larger quantity spot than they would forward--this is possible in cases 3-5. In these cases, however, the fully allocated unit profits from selling spot are negative. Although it is optimal to make the extra spot sales at the low price given that the capital costs are sunk, the initial decision to expand the capacity is ex-post regrettable. With forward contracting the decision to expand capacity in these events is avoidable.[4]

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4. This paper focuses exclusively upon the positive value to forward contracts and therefore Γ is always positive. Presumably these contracts impose constraints on future transactions which are costly and which might make Γ negative in some cases. A full treatment of the problem would measure both the strategic costs and benefits for an optimally designed contract. We consider our estimation of the benefits to be one element of this process and are pursuing research on modelling the strategic costs to long-term contracts

3. Natural Gas Take-or-Pay Contracts

As an example of the application of this model for the estimation of the value to a producer of a forward contract we use the take-or-pay contracts that are common in the natural gas industry. Typically, once a natural gas field has been discovered and simultaneous with the decision to develop the field the producer negotiates with prospective buyers long-term contracts under which a customer commits itself to pay each year for a given quantity of the gas. The buyer must pay for the gas to which it has committed itself whether or not it actually takes the gas, hence the name take-or-pay. These commitments usually run for a duration of fifteen to twenty years. In many contracts the exact quantity to be taken and the price to be paid in any given year may be described either by a complex formula or by more indefinite contingencies including potential regulatory changes.

The 'ex-post bargaining problem' is commonly recognized to be one primary motivation for the use of take-or-pay contracts in the natural gas industry. Its importance has historically been so great that almost all gas fields were financed using take-or-pay contracts that committed nearly 100% of the available gas. Flexibility consisted exclusively in the right to transfer delivery of a small portion of contracted quantities across a minimal number of years. Little interest existed therefore in precisely assessing the significance of the strategic value of the long-term contracts.

The situation has changed dramatically in recent years. Short term markets in natural gas now coordinate a significant portion of deliveries. Producers in the United States and Canada have found themselves studying the development of a field which by all traditional capital budgeting rules appears to be economically justifiable but for which they are unable to

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and the relation between the benefits and the costs.

negotiate successfully the traditional level of 'take' commitments on the part of buyers. Similar problems have arisen for producers targetting the western European market. Producers have therefore raised the question of whether or not the traditional rules-of-thumb for the minimal long-term contract quantity of gas needed to justify the installation of capacity need to be changed. The problem of assessing the significance of the strategic value of long-term contracts to a particular developer has therefore become important.

In this section we demonstrate the use of the auction model to assess the importance of these contracts for four natural gas fields. To illustrate how different features of the market impact the strategic value of the forward contracts, we make two comparisons using the auction model. The first comparison focuses upon the number of potential customers. We analyze the strategic value of forward contracts for the Venture natural gas field in eastern Canada and compare it with the strategic value of forward contracts for the development of a field in Alberta. The market for the gas from the Venture field consists of a very small set of users in New England. The gas from the Albertan field can be routed to a larger number of users in the midwestern and west coast United States. We will see that the model estimates the strategic value of forward contracts to be significantly greater for the Venture field than for the Albertan field. This occurs because the competition among the many potential buyers for the Albertan gas makes the capacity decision less important for the seller's bargaining power.

The second comparison focuses upon the size of the initial capital expenditures necessary to make the gas deliverable. We analyze the Troll field in Norway and the Soviet gas field in Urengoi. The Troll field reportedly requires relatively low capital expenditures. The Soviet gas requires large expenditures on pipelines and therefore larger initial capital expenditures to make the gas deliverable. The model calculates the strategic

value of forward contracts to be relatively modest for the Troll field while for the Soviet gas field the value of the forward contracts are significant.

3.1 The Number of Buyers: The Venture Field vs. Albertan Fields

To assess the strategic value of a take-or-pay contract for a field we first need to determine the parameter values for the variables of our model. These include the field characteristics: (i) size of the field, (ii) the per unit capital costs, and (iii) the per unit operating costs. These also include the market or buyer characteristics: (iv) the number of buyers and (v) the range of possible reservation values for each buyer. We will depend upon figures for the Venture gas field which are taken from Adelman et al. (1985) and use this data to choose the values for our parameters.[5]

Field characteristics. The Venture field has total reserves of 2.36 TCF (trillion cubic feet) and will be operated at a level of 116.8 BCF per year for a period of twenty years. We will analyze the value of contracts on an annual basis and therefore the total capacity used in the auction model will be this annual capacity. The capital expenditures necessary to develop the field are $1837.5 million. Amortized over the life of the field and the quantity to be delivered each year, these expenditures amount to $2.66/Mcf ($ thousand cubic feet). This will be the constant marginal capital cost figure used in the model. The operating costs are $75 million/year: the field, however, will produce associated liquids the sale of which will approximately equal the operating costs, and therefore we will set the operating costs per Mcf at zero.

Market/buyer characteristics. The gas from the Venture field will be

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5 The numbers and examples used in this paper are meant to illustrate the use of the auction model and are not meant as definitive analyses of the chosen projects, and we therefore do not detail the derivation of the data.

sold in the northeastern US market where there are a relatively small number of large buyers. Current sales by Canadian producers to this market are less than double the capacity of the Venture field. We model the negotiations as taking place between the field developer and three buyers, each of which could utilize the full annual capacity of the field: negotiations are therefore modelled as an auction of a single unit--116.8 Bcf/yr of gas--to three bidders. The range of prices to which these buyers might agree in a contract will be determined by the alternative sources of supply that are available to these buyers. Additional western Canadian gas might be available in the near future in this market at a price of approximately $2.90/Mcf at the border, and at $3.15 or $3.40/Mcf at the border in the next decade. Additional gas from Louisiana would be available delivered in Boston at $4.02/Mcf. Current consumption levels could perhaps be supplied from Louisiana at as low as $2.80/Mcf delivered. Transportation costs from the Venture field to the border and to Boston could be as high as 60 and 90 cents per Mcf respectively, although figures of 30 and 50 cents are perhaps more realistic. The range of wellhead prices which customers in the northeastern US are likely to accept may therefore lie between $2.30 and $3.52/Mcf, or could be as low as $2.00 to $3.10/Mcf. We use the former pair of prices to bound the range of buyer reservation values for our sample calculations in this paper.

These parameter values and the model calculations for expected profit from forward and spot contracting are displayed in Table 1.

[Insert Table 1]

The results of the model run for the Venture field can be interpreted as follows. If the developer of the Venture field enters into negotiations for forward commitments on the purchase of gas prior to the expenditure for development and pipeline construction, then the expected annual profit is $46 million. That totals $344 million NPV over the twenty year life of the

Table 1

The Strategic Value of Forward Contracts for the Venture Gas Field

Model Inputs:		Outputs, Net Present Value:	
Field Size	116.8 Bcf/yr: 20 yrs.	Forward Sales	$ 344 million
Capital Cost	$2.66 /Mcf	Spot Sales	$ 306 million
Operating Cost	$0.00 /Mcf	Difference (Γ)	$ 38 million
# of Buyers	3	% Change	-10.8
# of Sale Units	1		
Max. Res. Value	$3.52 /Mcf		
Min. Res. Value	$2.30 /Mcf		

project. If the developer were to install the capacity and subsequently attempt to sell the output 'spot', then the expected annual profits calculated by the model is $41 million or $307 million NPV over the life of the project.

These averages incorporate the possibility that in the negotiations the developer may find no buyer willing to commit itself to purchase the gas at the minimum price of $2.66/Mcf: the probability of this event is 3.7%. In the case of forward contracting the developer would cancel the project and incur no losses. In the case of spot sales the developer would have already made the capital expenditures and would sell the output at a price below the fully allocated cost, incurring a per unit loss. For an additional set of cases with a probability of 20%, the spot price covers capital and allocated costs, but it is still less than the forward price by about $0.15/Mcf. These two factors combine to make the profits expected from a strategy of development and spot sales $5 million less annually than the profits from a strategy of forward negotiations and development contingent on their outcome. This is a total net present value loss over the life of the project of $37 million or 10% of the NPV. This is the portion of the project NPV that is endangered in spot negotiations by the ex-post opportunism of the buyers: this is the portion of project NPV that is secured by means of forward contracts.

The Albertan Field characteristics. Data on the Albertan fields is also taken from Adelman et al. (1985). While the Venture data was derived from a field specific source, the Albertan field data is based upon typical costs for a class of fields in Alberta. We will model a field with a 20 year annual production capacity of 50 Bcf. The capital expenditures necessary to develop this field amount to $300 million over a three year period or an amortized expenditure based upon the planned rate of depletion of $1.01/Mcf. The operating expenditures necessary are $0.45/Mcf.

Market/buyer characteristics. The key difference between the Albertan

field and the Venture field is that the Albertan field can be connected into various pipeline networks which in turn each serve a broader number of large customers. Access is available both to the dense set of pipelines serving the US midwest and to pipelines serving the US west coast. For illustration we will model the problem as negotiations involving 5 large buyers each of which can consume the full output of the field: hence, the developer is negotiating to sell a single unit to one of five buyers. The range of reservation prices which we will use for each buyer is $1.00-4.00/Mcf, a range which generates an expected price of $3.32/Mcf. This price is comparable to that calculated in Adelman et al. (1985) as a likely scenario for exports from Alberta to the midwest and western US.

These parameter values and the model calculations for expected profit from forward and spot contracting are displayed in Table 2. Displays of the probability distributions of forward and spot prices and of the probability distributions of forward and spot profits generated by the auction model of negotiations for the Venture and Albertan fields appear in Figure 3.

[Insert Table 2 and Figure 3]

The results of the model run for an Albertan field can be interpreted as follows. If the developer of the Albertan field enters into negotiations for forward commitments on the purchase of gas prior to the expenditure for development and pipeline construction, then the model calculates the expected annual profits to be $186 million or $1.39 billion NPV over the life of the project. If the developer were to install the capacity and then attempt to sell the output 'spot', then the expected annual profits calculated by the model are also about $186 million--the exact difference is two-tenths of a percent of the annual profits. This is the portion of the project NPV which is endangered by the ex-post opportunism of the buyers and which is secured by means of forward contracts.

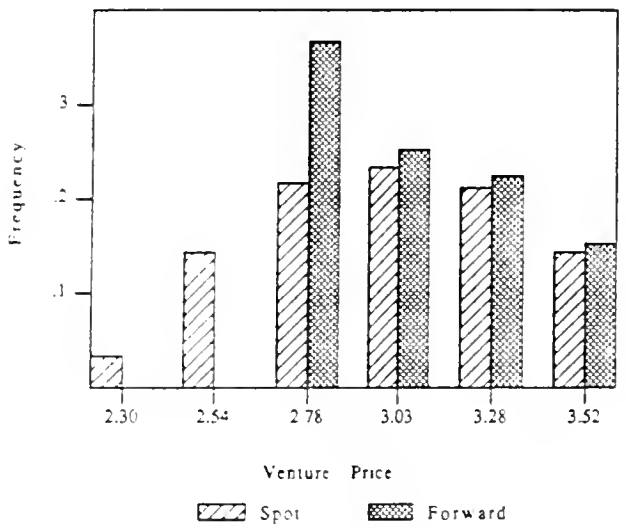
Table 2

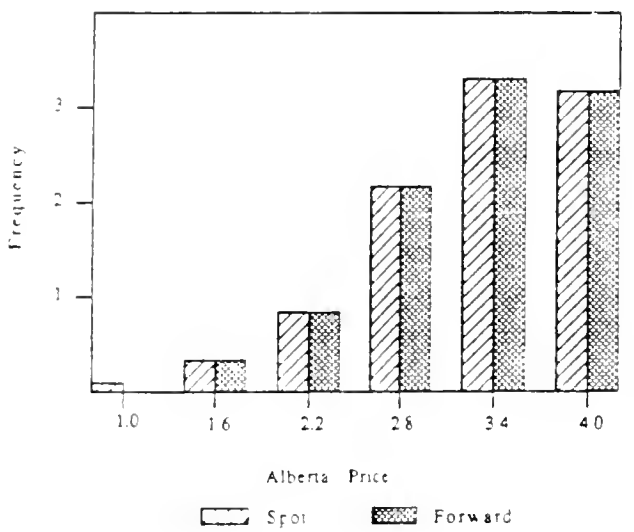
The Strategic Value of Forward Contracts for the Albertan Gas Field

Model Inputs:		Outputs, Net Present Value.	
Field Size	50 Bcf/yr; 20 yrs.	Forward Sales	$ 695 million
Capital Cost	$1.01 /Mcf	Spot Sales	$ 695 million
Operating Cost	$0.45 /Mcf	Difference (Γ)	$ 0 million
# of Buyers	5	% Change	- 0.2
# of Sale Units	1		
Max. Res. Value	$4.00 /Mcf		
Min. Res. Value	$1.00 /Mcf		

Figure 3a

Probability Distribution of Forward and Spot Prices
for the Venture and for the Albertan Gas Fields

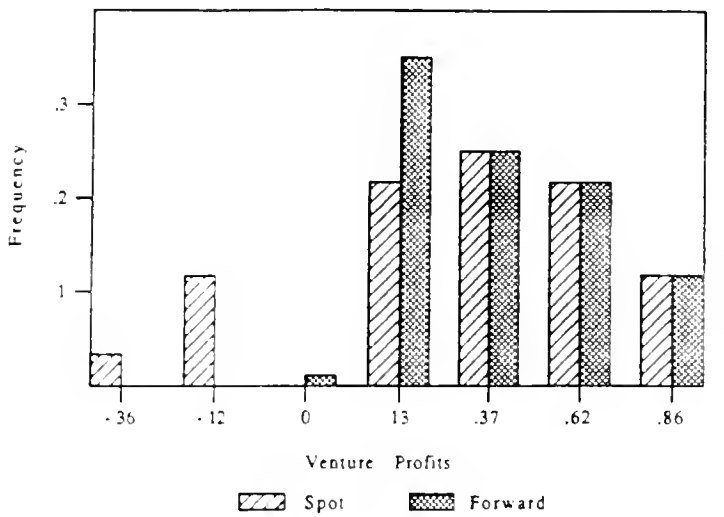


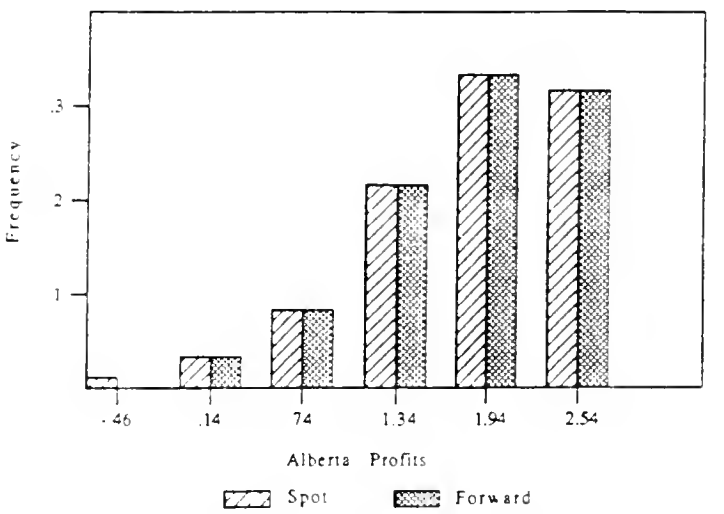


In the first figure or pair of distributions the darker of the two is the anticipated distribution of forward prices negotiated for the Venture gas field and the lighter of the two is the anticipated price distribution for spot sales of the Venture gas. In the second pair of distributions the forward and spot distributions for the Albertan field are displayed. In the case of Venture the forward price distribution is clearly shifted rightward. In the case of Alberta the forward and spot distributions are almost identical and hence the strategic value to forward contracts in the case of Albertan gas is virtually zero.

Figure 3b

Probability Distribution of Forward and Spot Unit Profits
for the Venture and for the Albertan Gas Fields





A comparison of the results displayed in Tables 1 and 2 illustrates the difference between the strategic value of the forward contracts for the Venture field and for the Albertan field. While the forward contracts secure for the producer more than 10% of the NPV of developing the Venture field, they are virtually irrelevant to the developer of a field in Alberta.

The forward contracts offer little strategic value in Alberta since with five bidders the probability is small that there are not at least two buyers with reservation prices above the marginal costs inclusive of capital charges, and therefore there is little probability that competition among the buyers will be absent leaving the seller dependent upon its bargaining power for ensuring a price sufficient to cover the capital and operating expenses. This can be seen in a comparison of the anticipated probability distributions of forward and spot prices and profits for the Venture field with the anticipated distributions for the Alberta field as displayed in Figure 3. The average profit from forward contracting for an Albertan field incorporates the possibility that in the negotiations the developer may find no buyer willing to commit itself to purchase the gas at the minimum price of $1.46/Mcf: the probability of this event is a mere 0.01% (contrast with 3.7% for Venture), and the probability of all events in which the spot price is less than the forward price is only 0.24% (contrast with 20% for Venture).

Assuming that we have captured the central motivation for the forward contracts, it would appear from our results that the gradual development of a 'thicker' market in natural gas in the midwest and western United States has significantly reduced for some fields the necessity of utilizing the strong take-or-pay contracts that have been common for the past several decades. Several persons in this industry have made such assertions, arguing that an increased reliance upon short term sales is possible: our model calculations support this claim. It is important to note, however, that this possibility

is restricted to particular areas: it would not, for example, be possible for gas to be marketed on this basis in New England, and as we shall see, is not possible for several other suppliers and markets in the world. The fact that these changes in the importance of contracts impact specific markets differentially is often ignored.

3.2 The Proportion of Capital Costs: Troll vs. Soviet Gas

Data on Norway's offshore Troll natural gas field is taken from Adelman et al. (1986). Total volume available from the Troll field is 14.2 Tcf or 610 Bcf per year for each of 23 years. Total capital expenditures for development will be $3.2 billion or $0.67/Mcf amortized over the schedule of production. Expenditures for pipeline construction will be another $2.56 billion--$0.62/Mcf. Per unit capital costs for the Troll field therefore amount to $1.29/Mcf. Operating expenses for the field will be $0.23/Mcf: operating expenses for the pipeline will be $0.18/Mcf. Total operating expenditures are therefore $0.41/Mcf.[6]

The gas from the Troll field will be piped into the western European market with France and the Federal Republic of Germany being the main buyers and potentially displacing gas to Italy. We will model the gas as being sold to three buyers. Each buyer is assumed to be able to completely purchase the scheduled annual output of 610 Bcf. The range of reservation values is based upon the demand profile provided in Adelman et al. (1986): $0.50-4.00/Mcf.

In the case of the Soviet Urengoi gas fields we will examine an expansion of production and transportation capacity of 1412 Bcf/yr producing for 20 years. Per unit capital costs for the Urengoi field are $1.59/Mcf. Per unit operating costs are $0.63/Mcf. The Soviet gas will be sold to the same

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6. The data for Troll used here originated in early published reports and may be relatively optimistic compared to later estimates.

markets that the Troll gas will be sold. We therefore model it as facing the same number of buyers and the same set of reservation values.

The parameter values and model calculations for these two fields are displayed in Table 3.

[Insert Table 3]

A comparison of the results for the Troll field with the results for the Urengoi field show the impact of the higher capital costs of the Urengoi field on the strategic value of the forward contracts. Given the same range of reservation values and the same number of buyers, the probability that there exist at least two buyers which value the resource at a price above the capital expenditures is greater for the field with the lower capital expenditures, Troll, than for the field with the higher capital costs. While it is immediate that the field with the lower costs per unit of production shows a higher profit margin when facing identical sets of buyers, our concern here is not with the absolute profit level, but with the percent of the margin which is secured via contracts. The model allows us to identify this

Again, one can see these results in a comparison of the forward and spot price and profit distributions for the two fields as displayed in Figure 4.

[Insert Figure 4]

4 Conclusion

Recent work in the economics of information has emphasized the role of long-term contracts in mitigating strategic problems which arise in imperfect or incomplete markets. In this paper we have shown that an auction model can be used to operationalize the results of this body of literature. Using the auction model we can estimate the significance of these strategic problems for a given project and the portion of the project's value which is secured to the

Table 3

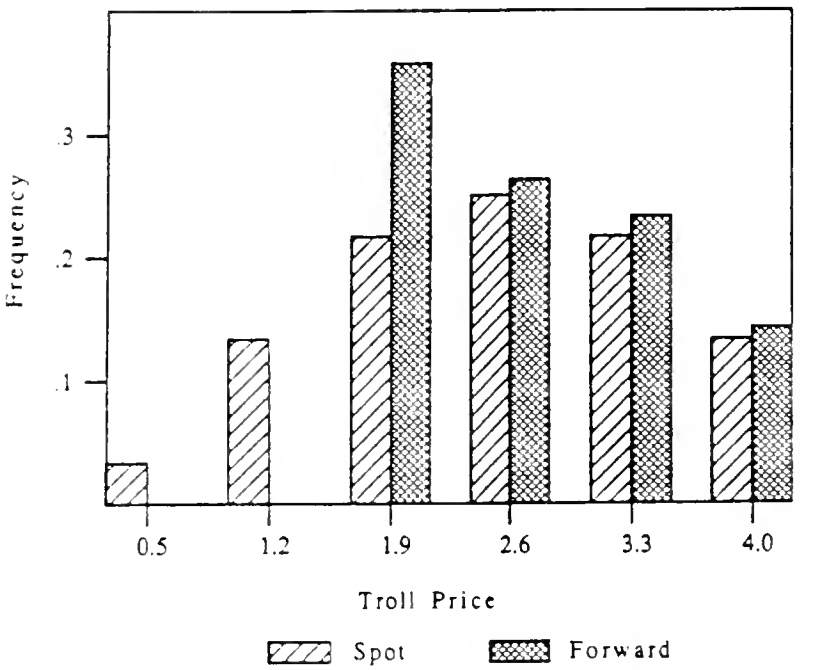
The Strategic Value of Forward Contracts for the Troll & Urengoi Gas Fields

Troll Model Inputs:		Outputs, Net Present Value:	
Field Size	610 Bcf/yr; 23 yrs.	Forward Sales	$4,571 million
Capital Cost	$1.29 /Mcf	Spot Sales	$3,965 million
Operating Cost	$0.41 /Mcf	Difference (Γ)	$ 606 million
# of Buyers	3	% Change	-13.3
# of Sale Units	1		
Max. Res. Value	$4.00 Mcf		
Min. Res. Value	$0.50 Mcf		

Urengoi Model Inputs:		Outputs, Net Present Value:	
Field Size	1412 Bcf yr, 20 yrs.	Forward Sales	$7,202 million
Capital Cost	$1.59 Mcf	Spot Sales	$3,572 million
Operating Cost	$0.63 /Mcf	Difference (Γ)	$3,630 million
# of Buyers	3	% Change	-50.4
# of Sale Units	1		
Max. Res. Value	$4.00 /Mcf		
Min. Res. Value	$0.50 Mcf		

Figure 4a

Probability Distribution of Forward and Spot Prices
for the Troll and Urengoi Gas Fields



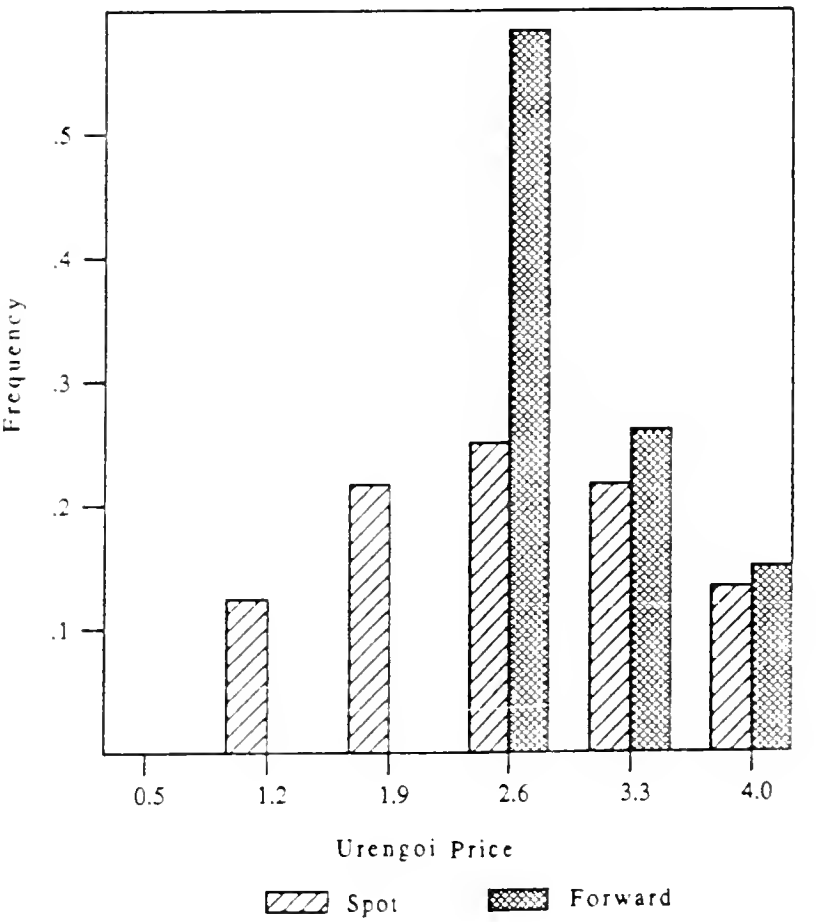
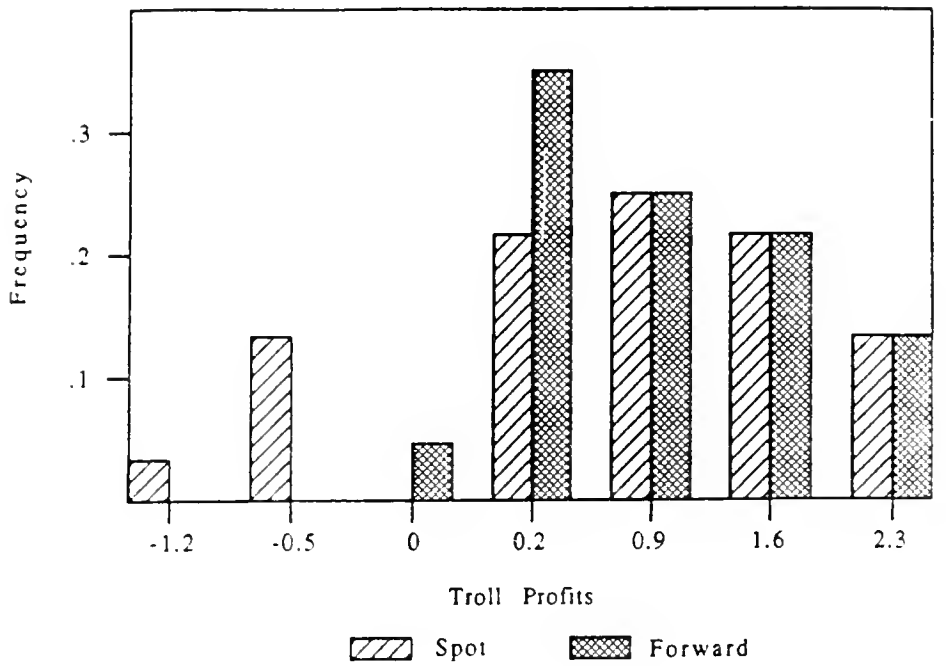
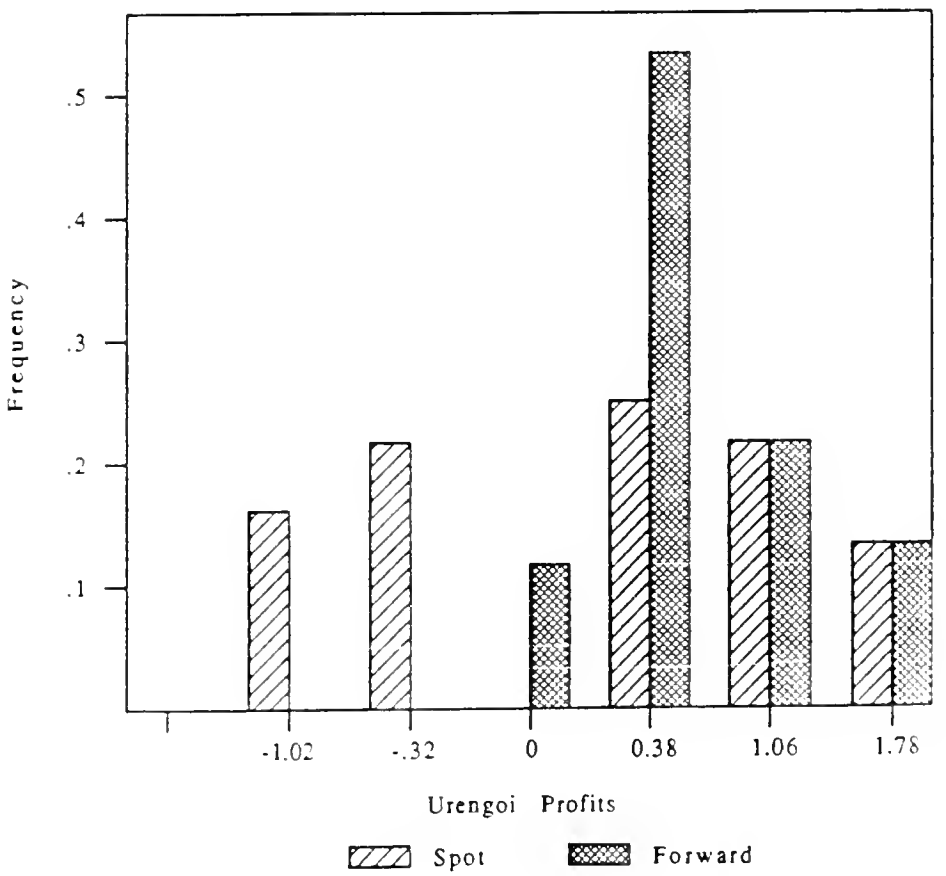


Figure 4b

Probability Distribution of Forward and Spot Unit Profits
for the Troll and Urengoi Gas Fields





producer by means of long-term contracts. This estimate allows us to incorporate strategic concerns into traditional and practical project valuation problems.

The strategic problems discussed in this paper are relevant for commodities which are traded in markets that are not perfectly competitive and for which large scale capital investments are necessary. The imperfect competition may be created in some cases when the capital investment is dedicated to a small set of buyers/sellers in an industry which otherwise includes a large number of buyers/sellers.

The strategic value to long-term contracts arises because the equilibrium prices negotiated in forward and spot contracts are influenced differently by the large scale capital investments made by the supplier. Use of forward contracts allows the capital investments to be made contingent upon the results of the negotiations. Spot contracts, on the other hand, are negotiated after capital has been irretrievably invested. The distribution of spot prices is typically biased downward due to the loss of bargaining power and under forward negotiations no sale is made when the costs inclusive of capital charges would be greater than the negotiated price. The auction model yields a consistent estimate of the consequences of these two factors based upon fundamental data on the size of the market, the demands of the buyers, and the cost structure for the industry. Other models typically used for analyzing forward contracts, such as the arbitrage technique applied by Brennan and Schwartz (1985) to long-term coal contracts, do not incorporate this key influence on the relationship between anticipated forward and spot distributions.

We have applied the auction model to the analysis of long-term take-or pay contracts used in the natural gas industry. While these long-term contract have been typical in this industry for decades we show, for example,

that in the midwest North American market the growing number of buyers to which a given seller can route their gas has significantly diminished the strategic value of these contracts. In contrast, where the number of available buyers remains small, as in gas routed to the New England market, the model yields a high strategic value to the traditional take-or-pay contract. In two cases from the European market we show how the strategic value to the contract also depends significantly upon the cost structure of the gas fields, with the strategic value diminishing as the proportion of the costs which must be incurred prior to spot negotiations falls.

Two important avenues of further research immediately propose themselves. First, the structure of the environment analyzed in this paper is extremely stylized: there is only one seller, the demand structures for all buyers are very simple and restrictive, as is the cost structure. On the one hand, these factors make optimal the use of very simple sale contracts containing no contingency or flexibility, and therefore allow us to view the problem with a minimum of complication. On the other hand, these restrictions limit the actual situations for which our model will yield accurate results. Moreover, these restrictions prevent us from making any analysis of or recommendations concerning the optimal design of more complicated forward contracts. We are currently analyzing the application of models of negotiations and bargaining in more complicated environments to the problem of long-term contracting.

The second area of further research follows from the fact that we have focused upon estimating the strategic factors which motivate or which favor the use of long-term contracts. We have left out of the analysis, for example, an estimation of the costs imposed upon both parties by the use of a forward contract. If, for example, production decisions should be made contingent upon information that arives after the capacity must be installed, and if the forward contract cannot easily incorporate this contingency because

the information is private to one party, then a forward contract will have strategic costs relative to the use of the spot market and these costs may outweigh the strategic value analyzed in this paper. The problem is properly posed as the design of interim efficient contracts in the sense of Holmstrom and Myerson (1983) and is the subject of our current research. The key variables which we are analyzing is the structure of optimal price indexes and the optimal design of interim 'take' or quantity decision rules.

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Appendix 1: Definition of Values used in the Modified Vickrey Auction

In accordance with Harris and Raviv (1981), to define the variables $(A_1,\ldots,A_{\ell-1})$ we first define three component variables.

$$a_i = \left[\frac{1}{k}\right]^{N-1} \sum_{j=Q}^{N-1} \sum_{\ell=0}^{Q-2} \binom{N-1}{j} \binom{j}{\ell} (k-i)^{\ell} (i-1)^{N-j-1} \qquad i=1,\ldots,k-1$$

$$b_i = \left[\frac{1}{k}\right]^{N-1} \binom{N-1}{N-Q} (k-i)^{Q-1} \left[i^{N-Q} - (i-1)^{N-Q}\right] \qquad i=1,\ldots,k-1$$

$$c_i = \left[\frac{1}{k}\right]^{N-1} \sum_{j=Q}^{N-1} \sum_{\ell=0}^{Q-1} \binom{N-1}{j} \binom{j}{\ell} (k-i)^{\ell} (i-1)^{N-j-1} \frac{Q-\ell}{j-\ell-1} \qquad i=1,\ldots,k-1$$

which may then be combined as $A_i = 1 - \frac{c_i - a_i}{b_i}$.

Appendix 2: Sample Displays of Auction Results and Calculation of Γ

In this section we present three examples of the modified Vickrey auction. The first example illustrates the rules of the auction with a complete listing of the price and quantity allocation for every combination of buyer reservation values in the feasible event space and the calculation of the expected profit. The second and third examples illustrate our calculation for the value of forward contracting. In the second example a complete listing of price and quantity allocations from an auction with the forward contracting minimum bid is displayed. In the third example a complete listing is given for identical parameters, but for an auction with the spot contracting minimum bid. The analysis of these two cases explicates the proof that the value to forward contracting is positive.

Example 1.

Table 4 below displays the full set of possible outcomes for an auction in which the manufacturer can produce a maximum of 1 unit at marginal fixed cost k=0.5 and marginal variable cost v=0, and in which there are 3 potential buyers, each with possible reservation values $(R_1,\ldots,R_4)=(0,1,2,3)$. The first column of the table indexes the possible events and the second column containing the vector of four numbers is a list of each possible combination of numbers of buyers at each of the four reservation prices; the third column is the frequency of that event given our assumed uniform distribution for each reservation value over the range of reservation prices; the fourth column lists the modified-Vickrey auction price that would follow for each event, and the fifth column lists the total quantity that would be sold, the sixth column lists the producer's total profits for each event, and the seventh column lists the probability weighted profits for each event. Displayed at the bottom of the seventh column is the sum of its entries, the expected profit to the producer from this auction sale of forward contracts

[Insert Table 4]

Table 4

Example 1: Display of Event Space of Possible Buyer Valuations, the Outcome of the Modified Vickrey Auction for each Event, and the Expected Total Profit

Event	# buyers w/ reserv. val. 0	1	2	3	prob.	price	total quantity sold	profit	prob. weighted profit
e	η_1	η_2	η_3	η_4	$g(\eta_1,\ldots,\eta_4)$	p_m	$\sum_{i=1}^{2} q_i$	π_e	$g\pi_e$
1	3	0	0	0	.01563	1	0	0	0
2	2	1	0	0	.04688	1	1	.5	.02
3	2	0	1	0	.04688	1	1	.5	.02
4	2	0	0	1	.04688	1	1	.5	.02
5	1	2	0	0	.04688	1	1	.5	.02
6	1	1	1	0	.09375	1.6	1	1.1	.10
7	1	1	0	1	.09375	1.6	1	1.1	.10
8	1	0	2	0	.04688	2	1	1.5	.07
9	1	0	1	1	.09375	2.5	1	2.0	.19
10	1	0	0	2	.04688	3	1	2.5	.12
11	0	3	0	0	.01563	1	1	.5	.01
12	0	2	1	0	.04688	1.6	1	1.1	.05
13	0	2	0	1	.04688	1.6	1	1.1	.05
14	0	1	2	0	.04688	2	1	1.5	.07
15	0	1	1	1	.09375	2.5		2.0	.19
16	0	1	0	2	.04688	3	1	2.5	.12
17	0	0	2	0	.01563	2	1	1.5	.02
18	0	0	2	1	.04688	2.5	1	2.0	.10
19	0	0	1	2	.04688	3	1	2.5	.12
20	0	0	0	3	.01563	3	1	2.5	.04
								Expected Profits	= 1.43

Table 4: The example has the following parameters. There is a maximum capacity of one unit, Q=1; the capital cost for installation of the capacity is 0.5/unit, k=.5; the operating costs are zero, v=0; there are three potential buyers, N=3, each with a reservation price which may be chosen from a range of four reservation values between 0 and 3, ℓ=4 and $\{R_1,\ldots,R_4\}=\{0,1,2,3\}$.

(Table 4 continued)

Definitions for the variables heading the columns in Table 4 are as follows:

$\eta_i = |\{r_j | r_j = R_i\}|$, the total number of buyers with valuation R_i.

$g(\eta_1, \ldots \eta_\ell) = \frac{N!}{\prod_{j=1}^{\ell} \eta_j!} \prod_{j=1}^{\ell} (\frac{1}{\ell})^{\eta_j}$, the probability of the event that there are η_1 buyers with reserv. value R_1, etc.

Profits for each event are calculated as the price minus the capital and operating costs per unit multiplied by the total quantity sold in that event: $\pi_e = \sum_{i=1}^{3} [p_\pi - (k+v)]q_i$, the profit for the event e.

Expected Profits, $\pi_f = \sum_{e=1}^{20} g\pi_e$

The increments to the price function for this example are, $(A_1, A_2, A_3) = (0.667, 0.556, 0.533)$

Referring to the display in Table 4 we will discuss several different possible outcomes for the set of buyer reservation values as a tool for explicating the properties of the auction model and its relation to the likely outcomes from contract negotiations:

Case 1. In the first event there is no buyer with a reservation value greater than zero: the producer will not agree to install capacity for a price less than the total marginal cost, 0.5, and therefore the quantity sold is zero.

Case 2. In the second, third and fourth events there is only one buyer with a reservation value greater than or equal to the total marginal cost and therefore there is no competition driving the price above the minimum: as stated in equation 6 when this is the case the price is set at the minimum reservation price above the marginal cost and therefore the modified-Vickrey price is one.

Case 3. In the fifth event there are two buyers with reservation values equal to one and the price is therefore competed up to one but the producer cannot charge a higher price.

Case 4. In the sixth event there is one buyer with a reservation value equal to two and one with a reservation value equal to one: competition in this event will always drive the price up atleast to one. The buyer with the higher reservation value may be forced in some cases to bid a price greater than one in order to obtain the unit of supply, and therefore for this event the expected price in the modified-Vickrey auction is set slightly above one.

Case 5. In the eighth event there are two buyers with reservation values of two and they therefore compete the price up to two.

Example 2.

Table 5 below displays the full set of possible outcomes for an auction in which the manufacturer can produce a maximum of 2 units at marginal fixed and variable costs k=.5 and v=0, in which there are 3 potential buyers, each with possible reservation values $(R_1,...R_4)=(0,1,2,3)$.

[Insert Table 5]

Example 2 is therefore identical with example 1 except that the capacity of the producer has been expanded from one unit to two units. We can compare examples one and two to see how the relationship between the capacity and the number of buyers affects the outcome of the auction. Events numbered 7 and 8 in both Tables 4 & 5 illustrate this relationship. In both of these events the number of buyers with reservation values greater than marginal cost is two. In Table 4 the resultant prices for these two events were 1.6 and 2. This was due to competition between the two buyers for the one unit of output In events 7 & 8 in Table 5, although there are two buyers with reservation values above marginal cost, there are also two units of capacity available for sale and therefore there is no competition driving the price up above the minimal reservation value greater than the cost of production, one.

Example 3.

Table 6 displays the results for this example with Q=2 under the assumption that the manufacturer installs the capacity and sells the output

Table 5

Example 2: Auction Results for Forward Contracting

Event	# buyers w/ reserv. val. 0 1 2 3				prob.	price	total quantity sold	profit	prob. weighted profit
e	η_1	η_2	η_3	η_4	$g(\eta_1,\ldots,\eta_4)$	p_m	$\sum_{i=1}^{2} q_i$	π_e	$g\pi_e$
1	3	0	0	0	.01563	1	0	0	0
2	2	1	0	0	.04688	1	1	.5	.02
3	2	0	1	0	.04688	1	1	.5	.02
4	2	0	0	1	.04688	1	1	.5	.02
5	1	2	0	0	.04688	1	2	1.0	.05
6	1	1	1	0	.09375	1	2	1.0	.09
7	1	1	0	1	.09375	1	2	1.0	.09
8	1	0	2	0	.04688	1	2	1.0	.05
9	1	0	1	1	.09375	1	2	1.0	.09
10	1	0	0	2	.04688	1	2	1.0	.05
11	0	3	0	0	.01563	1	2	1.0	.02
12	0	2	1	0	.04688	1	2	1.0	.05
13	0	2	0	1	.04688	1	2	1.0	.05
14	0	1	2	0	.04688	1.6	2	2.2	.10
15	0	1	1	1	.09375	1.6	2	2.2	.21
16	0	1	0	2	.04688	1.6	2	2.2	.10
17	0	0	3	0	.01563	2	2	3.0	.05
18	0	0	2	1	.04688	2	2	3.0	.14
19	0	0	1	2	.04688	2.7	2	4.4	.21
20	0	0	0	3	.01563	3	2	5.0	.08

$$\pi_f = \sum_{e=1}^{20} g\pi_e = 1.49$$

Table 5: The example has the following parameters. There is a maximum capacity of two units, Q=2; the capital cost for installation of the capacity is 0.5/unit, k=.5; the operating costs are zero, v=0; there are three potential buyers, N=3, each with a reservation price which may be chosen from a range of four reservation values between 0 and 3, ℓ=4 and $\{R_1,\ldots,R_4\}=\{0,1,2,3\}$. Definitions for the variables heading the columns are as defined for Table 4

spot. In Table 6, the sixth columns lists the operating profits for each event, i.e. the profits from the spot sale when the sunk capital costs are disregarded. In the seventh and eighth columns the fixed costs are allocated to each event and the net profits and probability weighted net profits for each event are calculated. The total expected profit is the sum of the entries in column eight and is displayed at the bottom of the column.

[Insert Table 6]

A comparison of the results from Table 5 with the results from Table 6 illustrates the difference in profits that a seller can anticipate from using forward versus spot contracts. For example, in events 2-4, there is only one buyer with a valuation above the marginal cost, k+v. When the firm is negotiating forward contracts, i.e. as displayed in Table 5, it sells only one unit and incurs the capital cost only for the installation of one unit of capacity. When the firm calculates the expected results from installing the capacity and negotiating spot sales, i.e. as displayed in Table 6, the firm has already incurred the capital cost of installing one unit of capacity and cannot in these cases earn a price which covers the fixed costs. In events 5-10, there are two buyers with valuations greater than the marginal costs of production, inclusive of capital cost. When the firm is negotiating forward contracts, i.e. as displayed in Table 5, it installs the capacity to sell to these buyers only because the buyers agree to a price greater than the costs of production. When the firm calculates the expected results from installing the capacity and negotiating spot sales, i.e. as displayed in Table 6, it has installed capacity and cannot therefore force the price up to cover the marginal costs of production; the price is determined exclusively by competition and in these events competition does not drive the price above the costs of production. In the remaining events the price is determined exclusively by competition both for forward contracting and for spot contracting, and therefore the results are identical for the two cases.

In Table 6 the expected profits from selling spot are calculated and then compared with the expected profits from selling forward as exhibited in Table 5. In this example the expected profits from the operations decline by more than 38% when the producer fails to secure the forward contracts for its output prior to incurring the capital costs, from 1.49 to 0.92.

Table 6

Example 3: Auction Results for Spot Sales

Event	η_1	η_2	η_3	η_4	$g(\eta_1,\ldots\eta_4)$	p_m	$\sum_{i=1}^{2} q_i$	Ψ_e	$g\Psi_e$	Ψ_e-2k	$g(\Psi_e-2k)$
1	3	0	0	0	.01563	0	2	0	0	-1	-.02
2	2	1	0	0	.04688	0	2	0	0	-1	-.05
3	2	0	1	0	.04688	0	2	0	0	-1	-.05
4	2	0	0	1	.04688	0	2	0	0	-1	-.05
5	1	2	0	0	.04688	0.6	2	1.2	.06	0.2	.01
6	1	1	1	0	.09375	0.6	2	1.2	.11	0.2	.02
7	1	1	0	1	.09375	0.6	2	1.2	.11	0.2	.02
8	1	0	2	0	.04688	0.6	2	1.2	.06	0.2	.01
9	1	0	1	1	.09375	0.6	2	1.2	.11	0.2	.02
10	1	0	0	2	.04688	0.6	2	1.2	.06	0.2	.01
11	0	3	0	0	.01563	1	2	2.0	.03	1	.02
12	0	2	1	0	.04688	1	2	2.0	.09	1	.05
13	0	2	0	1	.04688	1	2	2.0	.09	1	.05
14	0	1	2	0	.04688	1.6	2	3.2	.15	2.2	.10
15	0	1	1	1	.09375	1.6	2	3.2	.30	2.2	.21
16	0	1	0	2	.04688	1.6	2	3.2	.15	2.2	.10
17	0	0	3	0	.01563	2	2	4.0	.06	3	.03
18	0	0	2	1	.04688	2	2	4.0	.19	3	.14
19	0	0	1	2	.04688	2.7	2	5.4	.25	4.4	.21
20	0	0	0	3	.01563	3	2	6.0	.09	5	.09

$$\Psi_s = \sum_{e=1}^{20} g\Psi_e = 1.92$$

$$\pi_s = \Psi_s - 2k = .92$$

$$\Gamma = \pi_f - \pi_s = .57$$

$$\%\Delta = -38$$

Table 6: The example has the following parameters identical with those used in example 2. There is a maximum capacity of two units, Q=2; the capital cost for installation of the capacity is 0.5 unit, k=.5; the operating costs are zero, v=0; there are three potential buyers, N=3, each with a reservation price which may be chosen from a range of four reservation values between 0 and 3. ℓ=4 and $\{R_1,\ldots R_\ell\}=\{0,1,2,3\}$. Example 3 differs from example 2 only in that the determination of price and quantity allocation is for spot sales.

Definitions for the variables heading the columns are as defined for Table 4 except for the following addition:

$\Psi_e = \sum_{i=1}^{2} [p_m - v]q_i$ profits on operations, exclusive of capital expenditures

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